Circles

Geometry

Chapter 10

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Geometry*
  - *By Larson and Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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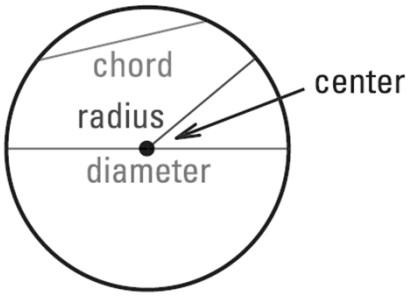
After this lesson…

- I can identify special segments and lines that intersect circles.
- I can use properties of tangents to solve problems.

10.1 Lines and Segments that Intersect Circles

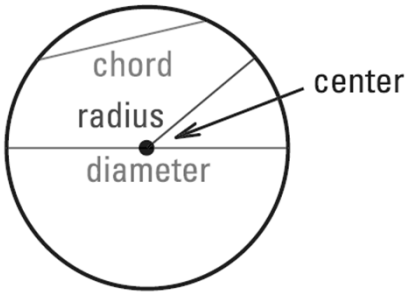
10.1 Lines and Segments that Intersect Circles

- Circle
  - All the points a given distance from a central point in a plane
  - Named by the center
- Radius (r) – the distance from the center of the circle to the edge.
- Chord – line segment that connects two points on a circle.



10.1 Lines and Segments that Intersect Circles

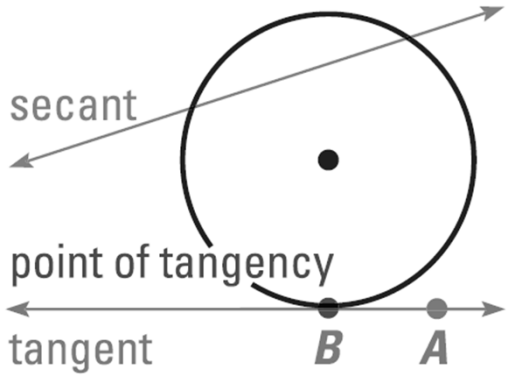
- Diameter (d) – chord that goes through the center of the circle (longest chord = 2 radii)
  - $d = 2r$
- What is the radius of a circle if the diameter is 16 feet?



$$d = 2r$$
$$16 = 2r$$
$$8 = r$$

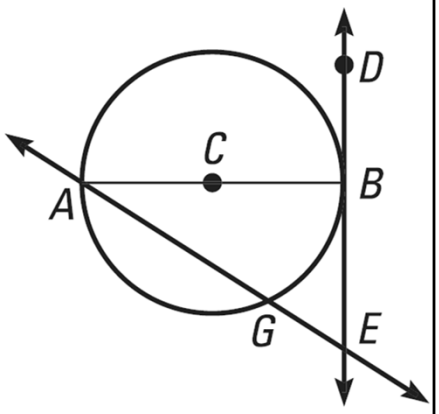
10.1 Lines and Segments that Intersect Circles

- Secant
  - Line that intersects a circle twice
- Tangent
  - Line that intersects a circle once



10.1 Lines and Segments that Intersect Circles

- What word best describes $\overline{AG}$?



- What word best describes $\overline{CB}$?

- Name a tangent and a secant.

- Try #6

Chord

Radius

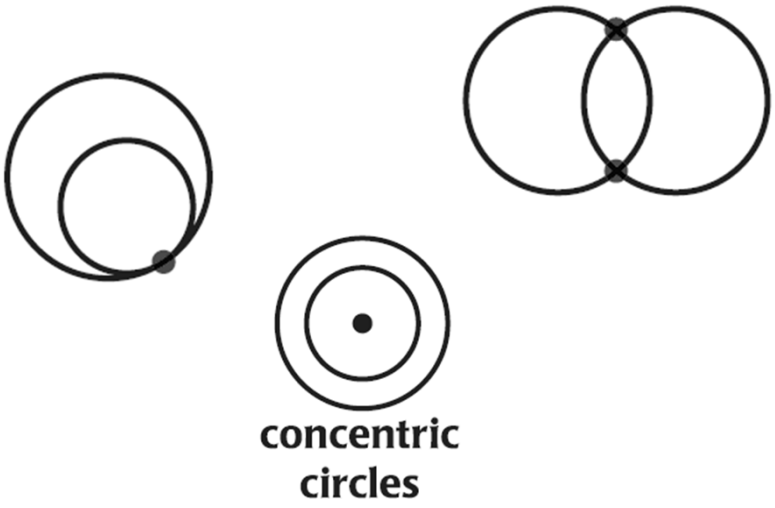
Tangent: $\overleftrightarrow{DE}$
Secant: $\overleftrightarrow{AG}$

10.1 Lines and Segments that Intersect Circles

- Two circles can intersect in 2 points

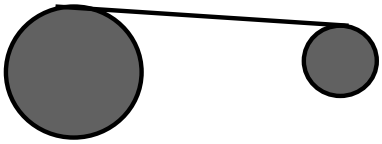
- 1 point

- No points

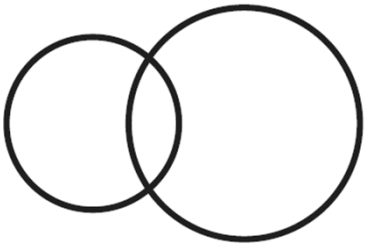


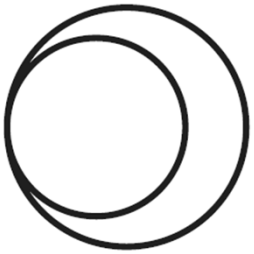
10.1 Lines and Segments that Intersect Circles

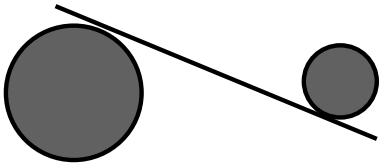
- Common tangents
  - Lines tangent to 2 circles

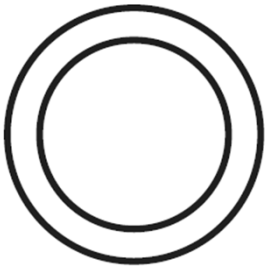


- How many common tangents do the circles have?









- Try #8

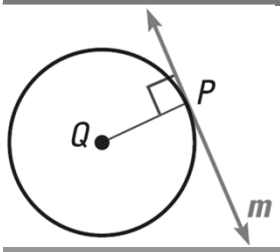
2

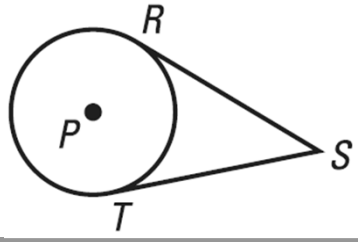
1

none

10.1 Lines and Segments that Intersect Circles

Tangent lines are perpendicular to radius.

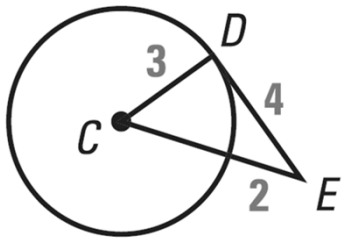




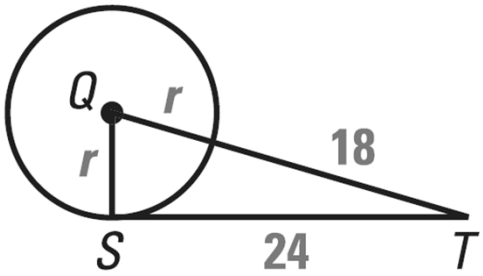
Tangent segments from the same point are congruent.

10.1 Lines and Segments that Intersect Circles

- Is $\overline{DE}$ tangent to $\odot C$?



- $\overline{ST}$ is a tangent to $\odot Q$. Find the value of r.



- Try #16

CE = 5

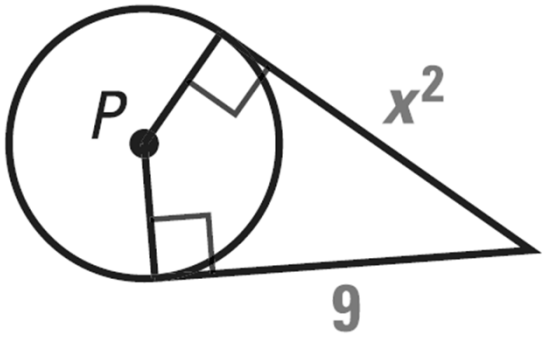
$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$
$$25 = 25$$

It makes a right angle, so CD ⊥ DE. Thus, DE is tangent

$$r^2 + 24^2 = (r + 18)^2$$
$$r^2 + 576 = r^2 + 36r + 324$$
$$576 = 36r + 324$$
$$252 = 36r$$
$$7 = r$$

10.1 Lines and Segments that Intersect Circles

- Find the value of x.



- Try #24

$$x^2 = 9$$
$$x = \pm 3$$

After this lesson…

- I can find arc measures.
- I can identify congruent arcs.

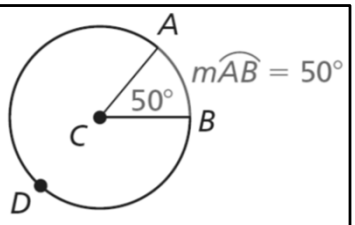
10.2 Finding Arc Measures

10.2 Finding Arc Measures

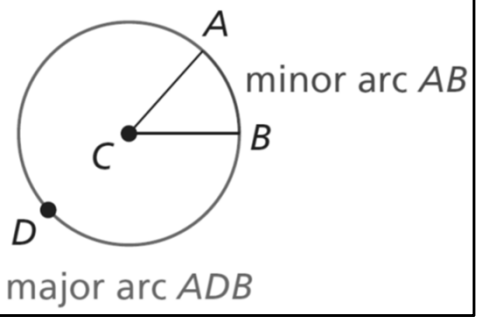
- How do you cut a pizza into eight equal pieces?
  - You cut in half, half, and half

- What measures are the angles in each piece?
  - 360 / 8 = 45°

10.2 Finding Arc Measures

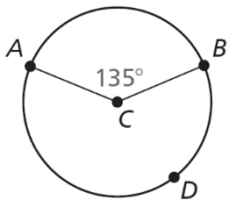


- There are 360° in a complete circle.
- Central Angle – Angle whose vertex is the center of the circle
- Arcs
  - An arc is a portion of a circle (curved line)
  - A central angle cuts a circle into two arcs
  - **Minor arc** – smaller of the two arcs – measures of arcs are the measures of the central angles
  - **Major arc** – bigger of the two arcs
  - Named $\overset{\frown}{AB}$ or $\overset{\frown}{ADB}$
    - use two endpoints to identify minor arc
    - use three letters to identify major arc



10.2 Finding Arc Measures

- Name the minor arc and find its measure. Then name the major arc and find its measure.



- Try #2

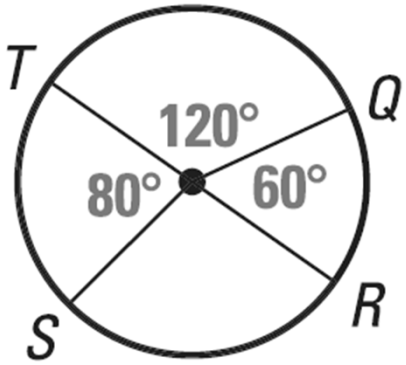
Minor: $\overset{\frown}{AB}$, 135°
Major: $\overset{\frown}{ADB}$, 225°

10.2 Finding Arc Measures

- Identify as major arc, minor arc, or semicircle. Find the measure.
- $\overset{\frown}{TQ}$

- $\overset{\frown}{TQR}$

- $\overset{\frown}{QRT}$



- Try #8

$\overset{\frown}{TQ}$ minor arc; 120°
$\overset{\frown}{TQR}$ semicircle; 180°
$\overset{\frown}{QRT}$ major arc; $m\overset{\frown}{RS} = 360° - 80° - 120° - 60° = 100°$

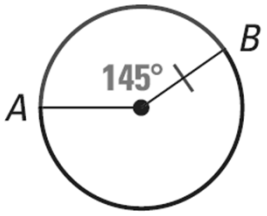
$$m\overset{\frown}{QRT} = 60° + 100° + 80° = 240°$$

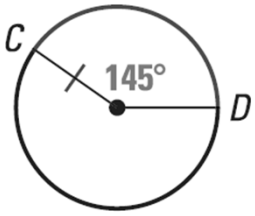
10.2 Finding Arc Measures

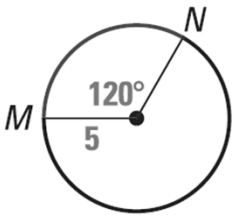
- Semicircle – arc if the central angle is 180°
- Similar Circles – all circles are similar
- Congruent circles – same radius
- Congruent arcs – same radius and measure

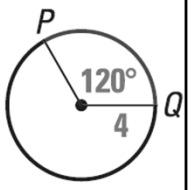
10.2 Finding Arc Measures

- Tell whether the red arcs are congruent.









- Try #16

Yes; same radius and angle

No; different radius

After this lesson…

- I can use chords of circles to find arc measures.
- I can use chords of circles to find lengths.
- I can describe the relationship between a diameter and a chord perpendicular to a diameter.

10.3 Using Chords

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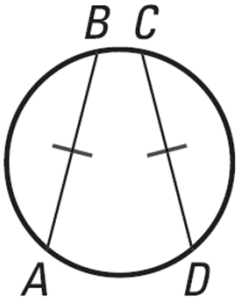
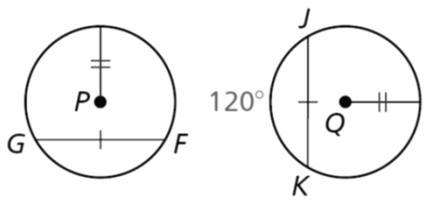
- Chords divide a circle into a major and minor arc.



In the same circle, or ≅ circles, two minor arcs are ≅ iff their chords are ≅.

10.3 Using Chords

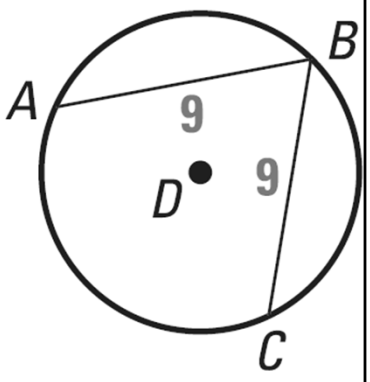
• Find $m\widehat{FG}$.



120° (Congruent chords of congruent circles)

10.3 Using Chords

- If $m\,\widehat{AB} = 110°$, find $m\,\widehat{BC}$.

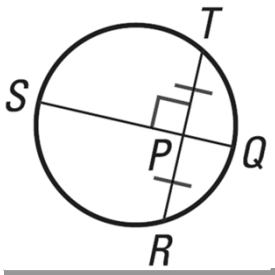


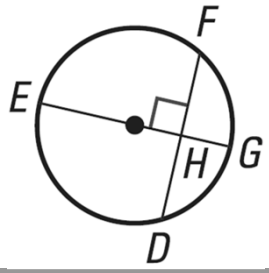
- Try #2

110°

10.3 Using Chords

If one chord is ⊥ bisector of another chord, then the 1st chord is diameter.

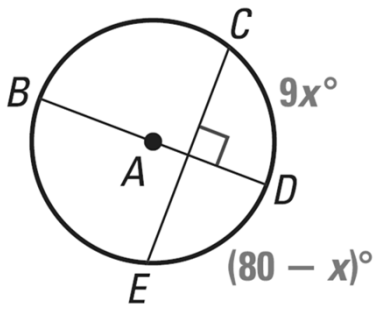




If a diameter is ⊥ to a chord, then it bisects the chord and its arc.

10.3 Using Chords

- Find the measure of the indicated arc.
- $\overset{\frown}{CD}$



- $\overset{\frown}{CE}$

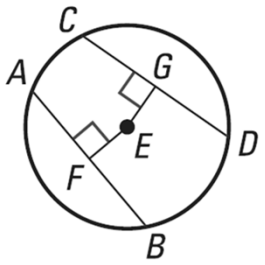
- Try #6

$$9x = 80 - x$$
$$10x = 80$$
$$x = 8$$
$$m\overset{\frown}{CD} = 9(8) = 72°$$

$$m\overset{\frown}{CE} = 2(72°) = 144°$$

10.3 Using Chords

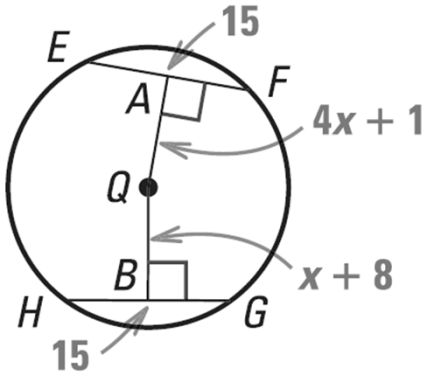
In the same ⊙, or ≅ ⊙, 2 chords are ≅ iff they are equidistant from the center.



$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

10.3 Using Chords

- Find the value of x.



- Try #14

$$4x + 1 = x + 8$$
$$3x + 1 = 8$$
$$3x = 7$$
$$x = \frac{7}{3}$$

After this lesson…

- I can find measures of inscribed angles and intercepted arcs.
- I can find angle measures of inscribed polygons.

10.4 Inscribed Angles and Polygons

10.4 Inscribed Angles and Polygons

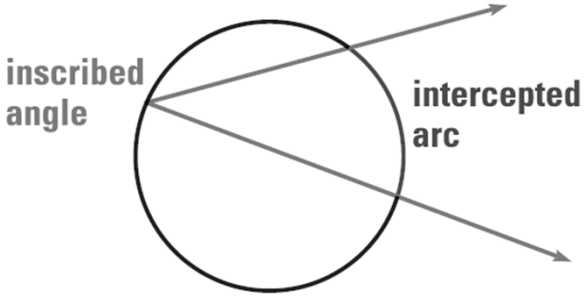
- What does inscribed mean?
  - Writing ON something; engraving ON

- Inscribed angle means the vertex ON the circle.

10.4 Inscribed Angles and Polygons

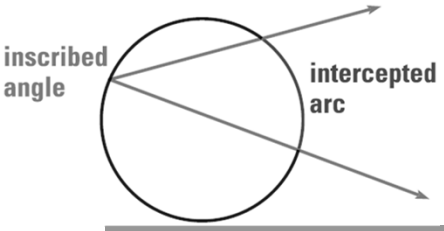
- Inscribed Angle
  - An angle whose vertex is on the edge of a circle and is inside the circle.

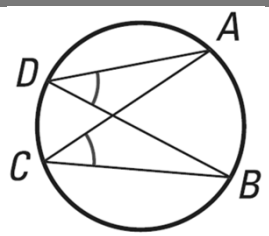
- Intercepted Arc
  - The arc of the circle that is in the angle.



10.4 Inscribed Angles and Polygons

The measure of an inscribed angle is ½ the measure of the intercepted arc.



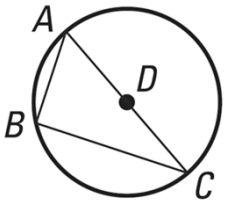


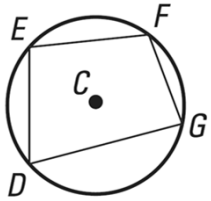
If two inscribed angles of the same or congruent circles intercept congruent arcs, then the angles are congruent.

10.4 Inscribed Angles and Polygons

If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle

½ 180 (semicircle) = 90

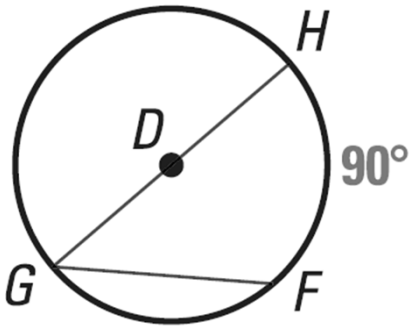


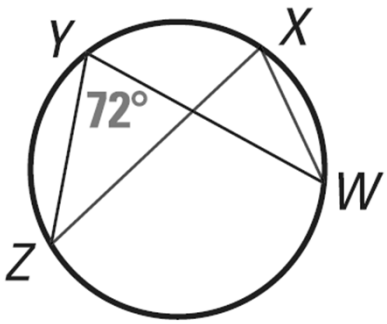


If a quadrilateral is inscribed in a circle, then the opposite angles are supplementary.

10.4 Inscribed Angles and Polygons

- Find the measure of the red arc or angle.





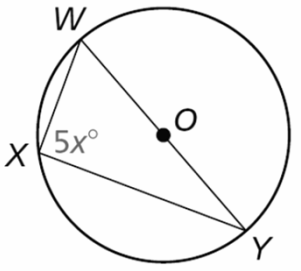
- Try #2

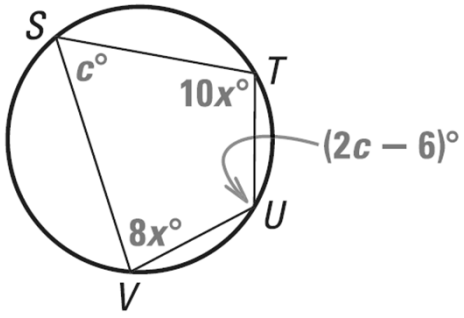
$$\frac{1}{2}90° = 45°$$

$$72°$$

10.4 Inscribed Angles and Polygons

- Find the value of each variable.





- Try #12
- Try #10

$$5x = 90$$
$$x = 18$$

$$8x + 10x = 180$$
$$18x = 180$$
$$x = 10$$
$$c + (2c - 6) = 180$$
$$3c - 6 = 180$$
$$3c = 186$$
$$c = 62$$

After this lesson…

- I can identify angles and arcs determined by chords, secants, and tangents.
- I can find angle measures and arc measures involving chords, secants, and tangents.
- I can use circumscribed angles to solve problems.

10.5 Angle Relationships in Circles

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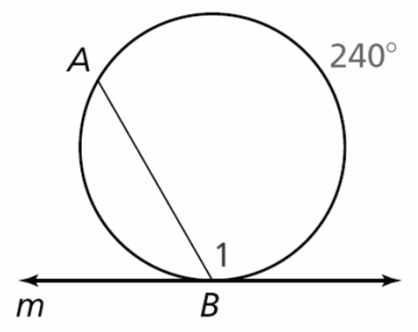
If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

- Find m∠1



- Try #2

$$\angle = \frac{1}{2}\text{inscribed arc}$$

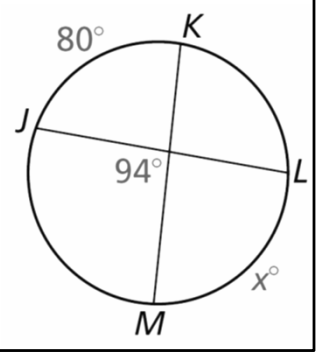
$$m\angle 1 = \frac{1}{2}(240°) = 120°$$

10.5 Angle Relationships in Circles

Angles Inside the Circle Theorem

If two secants intersect in the interior of a circle, then the measure of an angle formed is ½ the sum of the measures of the arcs intercepted by the angle and its vertical angle.

- Find the value of x.



- Try #6

$$\angle = \frac{1}{2}(arc1 + arc2)$$

$$angle\ between\ J\ and\ K = 180° - 94° = 86°$$

$$86° = \frac{1}{2}(x° + 80°)$$

$$172° = x + 80°$$

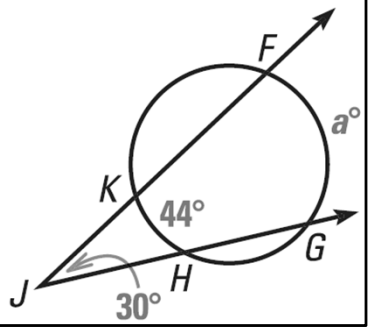
$$x = 92°$$

10.5 Angle Relationships in Circles

Angles Outside the Circle Theorem

If two secants, tangents, or one of each intersect in the exterior of a circle, then the measure of the angle formed is ½ the difference of the measures of the intercepted arcs.

- What is the value of a?



- Try #10

$$m\angle J = \frac{1}{2}\left(m\overset{\frown}{FG} - m\overset{\frown}{KH}\right)$$

$$30° = \frac{1}{2}(a° - 44°)$$

$$60° = a° - 44°$$

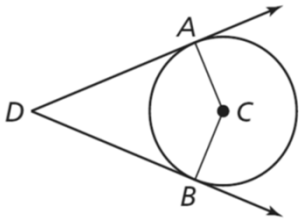
$$a = 104$$

10.5 Angle Relationships in Circles

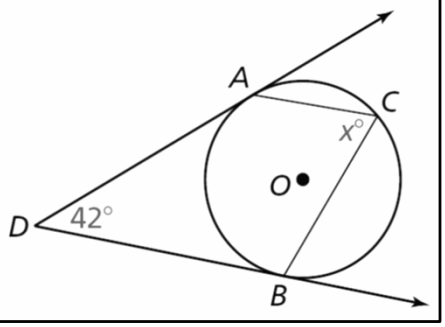
Circumscribed Angle Theorem

The measure of a circumscribed angle is equal to 180° minus the measure of the central angle that intercepts the same arc.

$$m\angle ADB = 180° - m\angle ACB$$



- What is the value of x?



$$x = \frac{1}{2} m\overset{\frown}{AB}$$

$$m\angle AOB = m\overset{\frown}{AB} = 2x$$

$$m\angle ADB = 180° - m\angle AOB$$

$$42° = 180° - 2x$$

$$-138° = -2x$$

$$69° = x$$

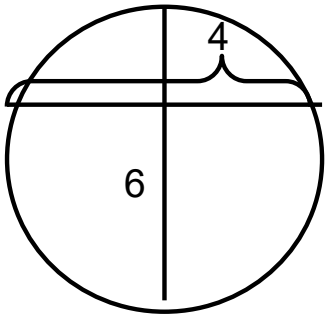
After this lesson…

- I can find lengths of segments of chords.
- I can identify segments of secants and tangents.
- I can find lengths of segments of secants and tangents.

10.6 Segment Relationships in Circles

10.6 Segment Relationships in Circles

- A person is stuck in a water pipe with unknown radius. He estimates that surface of the water makes a 4 ft chord near the top of the pipe and that the water is 6 ft deep. How much room is available for his head?



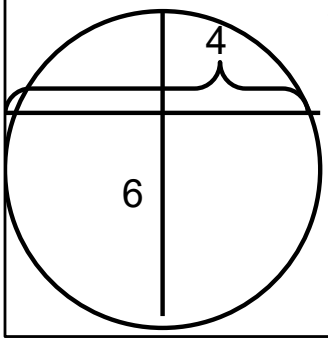
The chord can be subdivided into 2 ft and 2ft since the vertical line is a diameter. To answer the question we need to know the theorems in the section.

10.6 Segment Relationships in Circles

Segments of Chords Theorem

If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal.

- Take the example we started above.
  - The segments of the horizontal chords are 2 and 2; the segments of the vertical chords are 6 and x



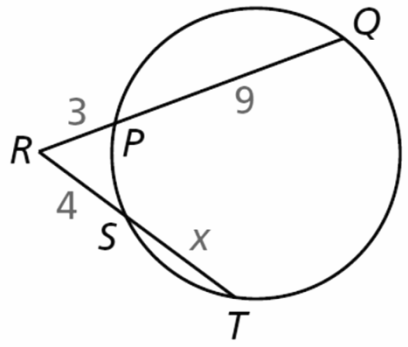
$$2(2) = 6x \quad x = \frac{4}{6} = \frac{2}{3}\ ft$$

Not much room for his head

10.6 Segment Relationships in Circles

Segments of Secants Theorem

If two secants are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.



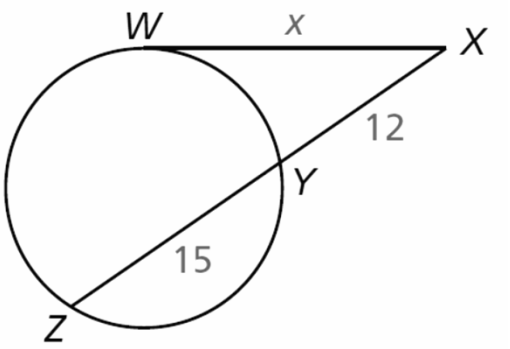
- Find x in the diagram.

$$(x + 4)4 = (12)3$$
$$(x + 4)4 = 36$$
$$x + 4 = 9$$
$$x = 5$$

10.6 Segment Relationships in Circles

Segments of Secants and Tangents Theorem

If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.



- Find x in the diagram

$$x^2 = (27)12$$
$$x^2 = 324$$
$$x = 18$$

After this lesson…

- I can write equations of circles.
- I can find the center and radius of a circle.
- I can graph equations of circles.
- I can write coordinate proofs involving circles.

10.7 Circles in the Coordinate Plane

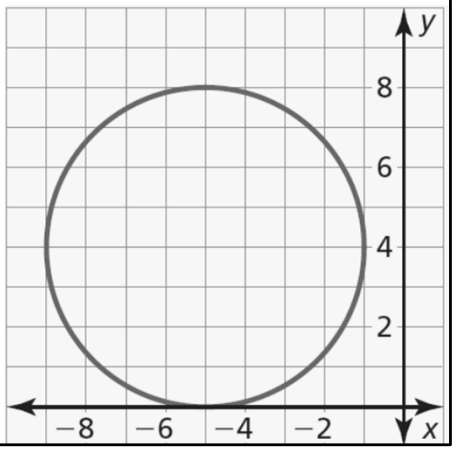
10.7 Circles in the Coordinate Plane

- Equation of a Circle

$$(x-h)^2+(y-k)^2=r^2$$

- Where (h, k) is the center and r is the radius

- Write the equation of the circle in the graph.



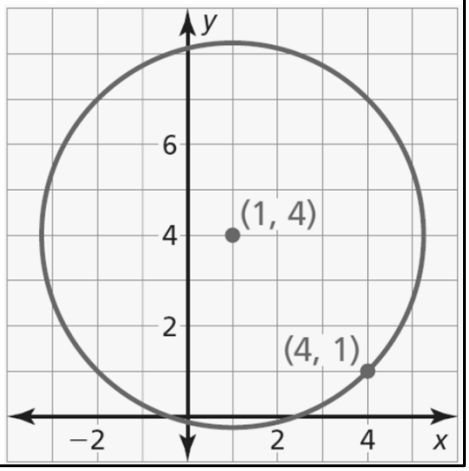
- Try #2

Center (-5, 4); h = -5; k = 4
Radius r = 4

$$(x-h)^2+(y-k)^2=r^2$$
$$\big(x-(-5)\big)^2+(y-4)^2=4^2$$
$$(x+5)^2+(y-4)^2=16$$

10.7 Circles in the Coordinate Plane

- Write the standard equation of the circle.



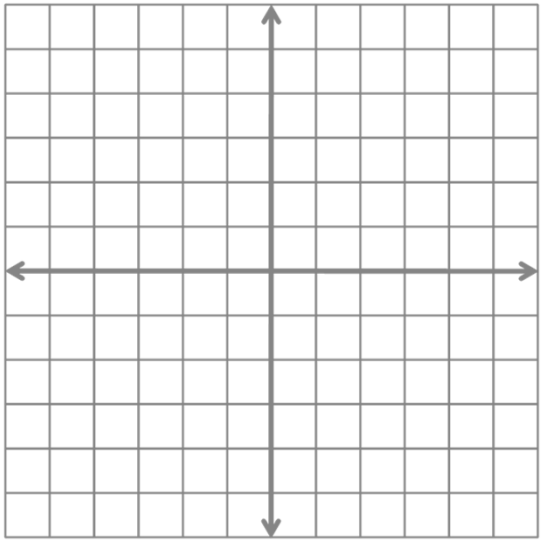
- Try #8

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$r = \sqrt{(1 - 4)^2 + (4 - 1)^2}$$
$$r = \sqrt{18} = 3\sqrt{2}$$
$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 1)^2 + (y - 4)^2 = \left(3\sqrt{2}\right)^2$$
$$(x - 1)^2 + (y - 4)^2 = 18$$

10.7 Circles in the Coordinate Plane

- Graph a circle by
  - Plot the center
  - Move every direction the distance r from the center
  - Draw a circle

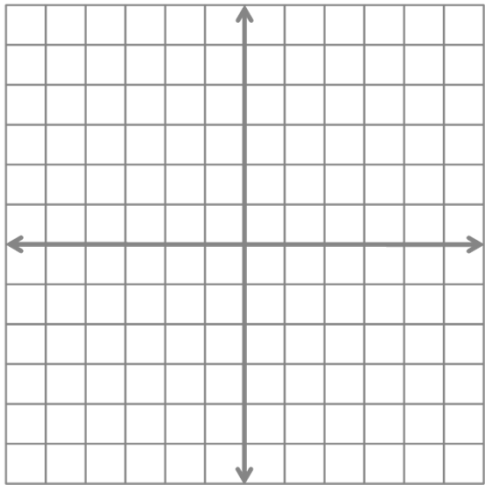
- Graph $(x-2)^2+(y+1)^2=4$



Center (2, -1)
$r^2 = 4 \rightarrow r = 2$

10.7 Circles in the Coordinate Plane

- The point (1, 4) is on a circle centered at the origin. Prove or disprove that the point $(3, \sqrt{7})$ is on the circle.



- Try #19

Center (0, 0)

$$r = \sqrt{(1-0)^2 + (4-0)^2} = \sqrt{17}$$
$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-0)^2 + (y-0)^2 = \sqrt{17}^2$$
$$x^2 + y^2 = 17$$

Try the point

$$(3)^2 + \left(\sqrt{7}\right)^2 = 17$$
$$9 + 7 = 17$$
$$16 \neq 17$$

The point is not on the circle